

# Robust Evolutionary Optimization Algorithm for Multi-objective Environmental/Economic Dispatch Problem with Uncertainties

1<sup>st</sup> José Nunes Rodrigues de Assis  
*Institute of Exact Sciences and Technology*  
*Federal University of Viçosa*  
Florestal, Brazil  
jose.n.assis@ufv.br

2<sup>nd</sup> Thiago Melo Machado-Coelho  
*Graduate Program in Electrical Engineering*  
*Federal University of Minas Gerais*  
Belo Horizonte, Brazil  
thmmcoelho@ufmg.br

3<sup>rd</sup> Gustavo Luís Soares  
*Graduate Program in Electrical Engineering*  
*Pontifical Catholic University of Minas Gerais*  
Belo Horizonte, Brazil  
gsoares@pucminas.br

4<sup>th</sup> Marcus Henrique Soares Mendes  
*Institute of Exact Sciences and Technology*  
*Federal University of Viçosa*  
Florestal, Brazil  
marcus.mendes@ufv.br

**Abstract**—The economic load dispatch problem has been formulated as the minimization of total fuel cost needed to generate electricity in power plants. Due to the environmental issues that arise from the emissions of polluting gases produced by fossil-fueled electric power plants, it becomes the environmental/economic dispatch (EED) multi-objective optimization problem. EED problems have been most commonly solved using deterministic models (without considering uncertainties). In this context, a robust model to cope with uncertainties is necessary, because the deterministic model is not able to reflect some real condition in practical applications, since fuel cost and emission coefficients may be subjected to inaccuracies, for instance. In this paper, both deterministic and robust EED problem models are first formulated, and the Robust Evolutionary Optimization Algorithm - REOA is proposed. For the robust model, the worst case scenario is estimated, to find reliable solutions that still feasible and have a good performance under the action of parametric uncertainties. Comparative studies are carried out to examine the effectiveness of the proposed approach. Simulation results are presented for the standard IEEE 30-bus system. The obtained results are satisfactory compared to the best solutions found in the literature.

**Index Terms**—Evolutionary Computation, Environmental/Economic Dispatch Problem, Robust Optimization, Multi-Objective Optimization

## I. INTRODUCTION

Nowadays there are serious environmental problems that urge for solutions. Reference [6] describes that environmental issues have had a greater impact on the economy due to the deterioration of the ecological environment. In terms of power generation, in general, a standard gas turbine plant with a thermal efficiency of 35% requires about 220 kg of natural gas to generate 1 MWh of electricity. The combustion of the fuel emits about 600 kg of carbon dioxide. For a plant with an average power capacity of 100 MW, the annual  $CO_2$  reduction would reach 5200 tonnes with a 1% reduction in

fuel consumption, which is possible by adopting appropriate measures of operational optimization and maintenance. In the world, for example, if all gas turbines have a generation capacity of  $4.5 \times 10^{12}$  kWh/year, the amount of  $CO_2$  that would cease to be emitted would exceed 25 million tons per year. Therefore, the study of operational optimization is necessary for the generation of energy in plants that emit polluting gases.

A classical environmental/economic dispatch (EED) problem is to operate electric power systems so as to minimize the total fuel cost and, at the same time, reduce the emissions produced by fossil-fueled electric power plants. It is a multi-objective optimization problem because pollution conflicts with the minimum cost of generation [7]. The EED problem has been formulated to determine the allocation of electric power for different generating units in order to minimize the total generation cost subject to technological and physical constraints. In recent years, with the increasing concern on environment protection, the EED problem has drawn much attention for reducing pollution [1]. It turns out to be a more desired power dispatch scheme when compared with the previous pure economic dispatch approaches. Various techniques have been proposed to solve this multi-objective problem whereby most researchers have concentrated on the deterministic problem without uncertainties. Therefore, it is necessary to construct a robust model, in which solutions remains feasible and have a good performance under consideration of parametric uncertainty.

After the introduction of environmental consideration in the economic dispatch problem, researchers started considering approaches that bear in mind uncertainties that are inherent in real systems. Reference [1] considers uncertainties as noise in the objective functions. Hence, a term in the objective

functions is added taking into account the mean and standard deviation of the output of each power generator unit of the plant. Reference [8] uses the Particle Swarm Optimization (PSO) algorithm and also considers uncertainties as noise in the objective functions. The objective functions were obtained through Taylor's series expansion around the mean, considering coefficients of variation of the random variables of power generator units. In this paper, we propose an evolutionary algorithm named Robust Evolutionary Optimization Algorithm (REOA), that provides robust solutions for multi-objective optimization problems considering parametric uncertainties. For the EED problem, REOA provides a different approach from those found in the literature, using parametric uncertainty and not a noise in the optimization functions.

In this paper, both the deterministic and robust models are addressed. More precisely, parametric uncertainties are considered in the robust model. The paper is organized as follows. The multi-objective and robustness concepts used in this paper are defined in Section 2. The EED problem is defined in Section 3. Section 4 outlines the system parameters considered in this study. Section 5 describes the REOA. The simulation results of the deterministic and robust models are given in Section 6. Based on these results, the main findings and some conclusions are outlined in Section 7.

## II. MULTI-OBJECTIVE OPTIMIZATION AND ROBUSTNESS

A multi-objective optimization problem can be formulated as follows [14]:

$$\begin{aligned} \min / \max \quad & \mathbf{f}(\mathbf{x}) = \{f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_{n_f}(\mathbf{x})\} \\ \text{Subject to} \quad & g_i(\mathbf{x}) \leq 0 \quad i = 1, \dots, n_g \\ & h_i(\mathbf{x}) = 0 \quad i = 1, \dots, n_h \\ & \mathbf{x} \in \mathbf{S} \end{aligned} \quad (1)$$

where  $\mathbf{f}(\mathbf{x})$  represents the objective functions vector,  $g_i(\mathbf{x})$  and  $h_i(\mathbf{x})$  the constraints functions,  $\mathbf{x} = (x_1, x_2, \dots, x_{n_v})^T$  the vector of decision variables and,  $\mathbf{S} \subseteq \mathbb{R}^{n_v}$  the feasible set.  $\mathbf{S}$  is given by:

$$\mathbf{S} = \{\mathbf{x} \in \mathbb{R}^{n_v} \mid \forall i \in \{1, \dots, n_g\} \wedge \forall j \in \{1, \dots, n_h\}, g_i(\mathbf{x}) \leq 0 \wedge h_j(\mathbf{x}) = 0\} \quad (2)$$

The objective functions are, for the most part, conflicting with each other [10]. In general, there is no single solution capable of optimizing all the objective functions simultaneously, but rather a set of solutions or Pareto set. The construction of a Pareto set is associated with the notion of Dominance. Considering minimization, a vector  $\mathbf{u} = (u_1, \dots, u_{n_f})$  is considered better than  $\mathbf{v} = (v_1, \dots, v_{n_f})$  if and only if  $\forall i \in \{1, \dots, n_f\}, u_i \leq v_i \wedge \exists i \in \{1, \dots, n_f\} \mid u_i < v_i$ . In words, we say that  $\mathbf{u}$  *dominates*  $\mathbf{v}$ , or  $\mathbf{u} \prec \mathbf{v}$ . According to [14], a vector  $\mathbf{x}^* \in \mathbf{S}$  belongs to the Pareto Set if there is not another vector  $\mathbf{x} \in \mathbf{S}$  such that  $f_i(\mathbf{x}) \leq f_i(\mathbf{x}^*) \forall i \in [1, 2, \dots, n_f] \wedge \exists j \in [1, 2, \dots, n_f] \mid f_j(\mathbf{x}) < f_j(\mathbf{x}^*)$  for at least one index  $j$ .

Multi-objective optimization problems are also subject to uncertainties, which are often difficult or impossible to avoid

in practice [5]. Reference [11] affirms that, in fact, uncertainty is present in several situations, such as, for example, when data is missing or corrupted, when the laws describing a phenomena are not completely known, or when the environment affects the system. According to [9], a robust solution can be defined as a solution that has satisfactory performance under parametric variations, not being sensitive to small variations of project or environment variables.

Considering the previous discussions and adding the uncertainty parameter  $\mathbf{p} \in \mathbf{P} \subseteq \mathbb{R}^{n_p}$ , the minimization robust multi-objective problem can be written as:

$$\min_{\mathbf{x} \in \mathbf{S}} \max_{\mathbf{p} \in \mathbf{P}} \mathbf{f}(\mathbf{x}, \mathbf{p}) = \{f_1(\mathbf{x}, \mathbf{p}), \dots, f_{n_f}(\mathbf{x}, \mathbf{p})\} \quad (3)$$

with the feasible region given by:

$$\mathbf{S} = \{\mathbf{x} \in \mathbf{X} \mid \mathbf{g}(\mathbf{x}, \mathbf{p}) \leq \mathbf{0} \wedge \mathbf{h}(\mathbf{x}, \mathbf{p}) = \mathbf{0}, \forall \mathbf{p} \in \mathbf{P}\}. \quad (4)$$

Solving (3) consists in finding the Pareto set of *robust minimizers*  $\mathbf{X}^*$ ,

$$\mathbf{X}^* = \{\mathbf{x}^* \in \mathbf{S} \mid \nexists \mathbf{x} \in \mathbf{S}, \max_{\mathbf{p} \in \mathbf{P}} \mathbf{f}(\mathbf{x}, \mathbf{p}) \prec \max_{\mathbf{p}' \in \mathbf{P}} \mathbf{f}(\mathbf{x}^*, \mathbf{p}')\} \quad (5)$$

Thus, the concept of robustness used in this paper is to discover solutions that remain efficient and feasible for all specified levels of uncertainty, even when exposed to small disturbances. Such solutions are called robust solutions.

## III. ENVIRONMENTAL/ECONOMIC DISPATCH

The environmental/economic dispatch involves the simultaneous optimization of fuel cost and  $CO_2$  emission. Nonetheless, there are uncertainties in power system operations, the typical formulation of planning and scheduling power generation activities remains deterministic. The uncertainties may come from internal parameter changes as well as external continuously varying factors such as load demands. The deterministic model and its robust version are formulated as described below.

### A. Objective Functions

a) *Fuel Cost*: The deterministic model for the classical economic dispatch problem of finding the optimal combination of power generation, which minimizes the total fuel cost while satisfying the total required demand, can be mathematically stated as follows according to [1]:

$$C = \sum_{i=1}^n (a_i + b_i x_i + c_i x_i^2) \quad \$/\text{hr}, \quad (6)$$

where:

- $C$ : total fuel cost per hour (\$/hr),
- $a_i, b_i, c_i$ : fuel cost coefficients of generator  $i$ ,
- $x_i$ : power generated in power unit (p.u.) by generator  $i$ , and,
- $n$ : number of generators.

The robust model is obtained by adding uncertainties in the decision variables. Thus, the expected fuel cost can be expressed as follows:

$$C = \sum_{i=1}^n (a_i + b_i[x_i + p_i] + c_i[x_i + p_i]^2) \quad \$/\text{hr}, \quad (7)$$

where  $p_i$  is the uncertainty relative to power generator  $i$ .

b)  $NO_x$  Emission: The amount of pollutants emission, which can be modeled using second order polynomial functions [1]:

$$E_{NO_x} = \sum_{i=1}^n (\alpha_i + \beta_i x_i + \gamma_i x_i^2 + \xi_i \sin(\delta_i x_i)) \quad \text{ton/hr}, \quad (8)$$

where:

- $E_{NO_x}$ :  $NO_x$  emission in tons per hour (ton/hr),
- $\alpha_i, \beta_i, \gamma_i, \xi_i, \delta_i$ : polluting emission coefficients of each generator  $i$ ,

Thus, the expected emission for the robust model can be achieved by the form:

$$E_{NO_x} = \sum_{i=1}^n (\alpha_i + \beta_i[x_i + p_i] + \gamma_i[x_i + p_i]^2 + \xi_i \sin(\delta_i[x_i + p_i])) \quad (9)$$

### B. Constraints

The deterministic and robust optimization models are bounded by the following constraints:

a) *Power Balance Constraint*: The total power generated must supply the total load demand and the transmission losses.

$$\sum_{i=1}^n X_G - X_D - X_L = 0, \quad (10)$$

where:

- $X_G$ : total power generated (p.u.),
- $X_D$ : total load demand (p.u.), and
- $X_L$ : transmission losses (p.u.).

b) *Maximum and Minimum Limits of Power Generation*:

The power  $x_i$  generated by each generator is constrained between its minimum and maximum limits, for the deterministic model by the form:

$$x_{iMin} \leq x_i \leq x_{iMax} \quad (11)$$

For the robust model, the following statement is used:

$$x_{iMin} \leq x_i + p_i \leq x_{iMax} \quad (12)$$

The network losses are modeled using Kron's loss formula as follows:

$$X_L = \sum_{i=1}^n \sum_{j=1}^n (x_i + p_i) B_{ij} (x_j + p_j) + \sum_{i=1}^n B_{0i} (x_i + p_i) + B_{00}, \quad (13)$$

where  $B_{ij}$ ,  $B_{0j}$  and  $B_{00}$  are the loss coefficients (or B-coefficients) listed below:

$$B_{ij} = \begin{bmatrix} 0.02180 & 0.01070 & -0.00036 & -0.00110 & 0.00055 & 0.00330 \\ 0.01070 & 0.01704 & -0.00010 & -0.00179 & 0.00026 & 0.00280 \\ -0.00040 & -0.00010 & 0.02459 & -0.01328 & -0.01180 & -0.00790 \\ -0.00110 & -0.00179 & -0.01328 & 0.02650 & 0.00980 & 0.00450 \\ 0.00055 & 0.00026 & -0.01180 & 0.00980 & 0.02160 & -0.00010 \\ 0.00330 & 0.00280 & -0.00790 & 0.00450 & -0.00010 & 0.02978 \end{bmatrix} \quad (14)$$

$$B_{0i} = 10^{-3} \times [0.010731, 1.7704, -4.0645, 3.8453, 1.3832, 5.5503] \quad (15)$$

$$B_{00} = 0.0014 \quad (16)$$

### C. Uncertainties

Since REOA uses parametric uncertainty, the way used to calculate the interval is determined by the following equation:

$$p_i = 0.03(x_{iMax} - x_{iMin}). \quad (17)$$

$p_i$  indicates that the uncertainty associated with generator  $i$  will be in the interval  $[-p_i, p_i]$ . That is, each decision variable can have up to 3% more or less disturbance of its original value.

### IV. SYSTEM PARAMETERS

Simulations were performed on the standard IEEE 30-bus 6-generator test system, shown in Fig. (1), using the Elitist Nondominated Sorting Genetic Algorithm (NSGA-II) for the deterministic model and REOA for the robust model.

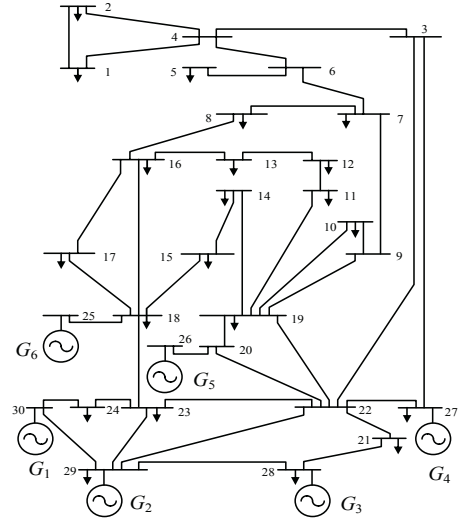


Fig. 1. IEEE 30-bus power test system.

The power system is interconnected by 41 transmission lines and the total system demand for the 21 load buses is 2.834

power unit (p.u). The coefficients of fuel costs and emissions used in simulations are given in Table (I) and Table (II), respectively.

TABLE I  
FUEL COST COEFFICIENTS AND GENERATOR CAPACITIES

Generator $i$	$x_{iMin}$	$x_{iMax}$	$a_i$	$b_i$	$c_i$
1	0.05	0.50	10	200	100
2	0.05	0.60	10	150	120
3	0.05	1.00	20	180	40
4	0.05	1.20	10	100	60
5	0.05	1.00	20	180	40
6	0.05	0.60	10	150	100

TABLE II  
POLLUTANTS EMISSION COEFFICIENTS

Generator $i$	$\alpha_i$	$\beta_i$	$\gamma_i$	$\xi_i$	$\delta_i$
1	4.091e-2	-5.554e-2	6.490e-2	2.0e-4	2.857
2	2.543e-2	-6.047e-2	5.638e-2	5.0e-4	3.333
3	4.258e-2	-5.094e-2	4.586e-2	1.0e-6	8.000
4	5.326e-2	-3.550e-2	3.380e-2	2.0e-3	2.000
5	4.258e-2	-5.094e-2	4.586e-2	1.0e-6	8.000
6	6.131e-2	-5.555e-2	5.151e-2	1.0e-5	6.667

## V. ROBUST EVOLUTIONARY OPTIMIZATION ALGORITHM - REOA

The proposed Robust Evolutionary Optimization Algorithm, REOA, is based on the NSGA-2 algorithm, and uses worst case estimation to solve multi-objective optimization problems with uncertainties.

### A. Nondominated Sorting Genetic Algorithm - NSGA II

NSGA-II [2] works basically as a common genetic algorithm, but some steps are added, such as the nondominated sorting and crowding distance for all the solutions. In the nondominated sorting step, all solutions are divided in fronts, according to the dominance of each solution. The solutions in the first nondominated front will have their domination count as zero, which means there is no solution that dominates any solution in the front. The second nondominated front will have solutions with the domination count as one, which means those solutions are dominated only by solutions in the first front. Solutions in the third front are only dominated by solutions in the first two fronts, and this is repeated until all solutions are sorted in fronts. For the crowding distance step, the objective is to reduce the density of solutions in each front, which will result in more variety of the population, since solutions that are very close to each other one of will be eliminated. More details about NSGA-II may be found in [2].

A flowchart of the REOA can be seen in Fig. (2). The first step is to generate the initial population. The second step is to calculate the fitness of each solution using the proposed robust concept. The third step is to check if the stopping criterion is achieved, if it is not, the algorithm will continue. The fourth step, if the stopping criterion was not achieved, is to apply the crossover operation. The fifth step is the mutation

operation. The sixth step is to calculate the fitness of each solution as in the second step. The seventh step is to perform the nondominated sorting. Finally, the eighth step is calculate the crowding distance of the solutions in each front created. Afterward, the algorithm go back to step three. If the stopping criterion is achieved, the execution is stopped and the Pareto frontier is returned.

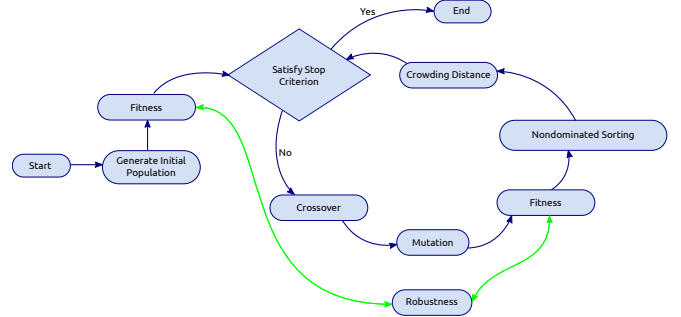


Fig. 2. Flowchart of the REOA

### B. Worst Case Estimation - WCE

REOA uses a minimax strategy, in which the worst case of uncertainties is found by a process of internal maximization of a minimization problem. The minimax strategy consists of a procedure of high computational complexity, requiring a lot of time to solve optimization problems. To make this strategy viable, a worst case estimation technique proposed by [3] was used. According to [3], if the function  $f_i(x)$  under analysis is monotonic in  $U(x)$  with respect to all decision variables, or convex, the worst case of uncertainties around a nominal solution  $x_0$  and the worst value of restriction will occur at one of the vertices of the domain  $U(x_0)$  for the uncertainty vector  $P$ .

The evaluation of the  $2^{n_v}$  vertices of each  $f(x)$  (black circles in Fig. (3)) may be computationally infeasible for problems of high dimension  $n_v$  (number of decision variables). Thus, rather than trying to find the worst case exhaustively, it is estimated which is the vertex of the worst case of uncertainties. According to [3], it is possible to estimate the worst-case uncertainty vertex with cost  $2n_v + 1$  evaluations, which is much lower than the cost of  $2^{n_v}$  evaluations.

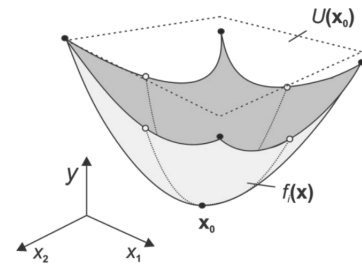


Fig. 3. Example of convex function in the domain  $U(x_0)$  [3].

This technique proposed by [3] consists of evaluating the effect of interval limits of uncertainties on each dimension  $n_v$

of the problem for each objective function. The interval limit that generates the worst result in a given dimension provides useful information of the direction of the vertex of the worst case of uncertainties in that dimension. According to [3] the formulation to estimate the worst case  $f_{wci}(x_0, P)$  is:

$$f_{wci}(x_0, P) = \max[(f_i(x_0, p^*), f_i(x_0))] \quad |$$

$$p^* = \begin{bmatrix} \text{sign}(f_i(x_0, \bar{p}_1) - f_i(x_0, \underline{p}_1))\Delta_1 \\ \text{sign}(f_i(x_0, \bar{p}_2) - f_i(x_0, \underline{p}_2))\Delta_2 \\ \vdots \\ \text{sign}(f_i(x_0, \bar{p}_{n_p}) - f_i(x_0, \underline{p}_{n_p}))\Delta_{n_p} \end{bmatrix}, \quad (18)$$

$$p^* \in P, i = 1, 2, \dots, n_f$$

where:

$$\Delta_j = \bar{p}_j, \quad \text{if } \text{sign}(f_i(x_0, \bar{p}_j) - f_i(x_0, \underline{p}_j)) > 0$$

$$\Delta_j = \underline{p}_j, \quad \text{otherwise.} \quad (19)$$

$$j = 1, 2, \dots, n_p.$$

The signum function  $\text{sign}(\Delta f)$  returns the signal of  $\Delta f$ , and is responsible for determining for each  $f_i$  in the worst case uncertainty direction. The value of  $\Delta_j$  represents the magnitude of the pitch toward the worst case of uncertainty for the  $i$ -th dimension, and assumes one of the limits of the perturbation, whether higher than  $\bar{p}_j$ , or lower than  $\underline{p}_j$ . After  $2n_v$  evaluations are made to determine the vector  $\bar{p}^*$ , then  $f_i(x_0, p^*)$  is calculated to definitively estimate the worst case, thus having  $2n_v + 1$  evaluations and the method has  $2n_v + 2$  evaluations, since (18) evaluates the original objective function without disturbance. This formulation is restricted to the parametric uncertainty, which affects only the decision variables. If the function  $f_i(x)$  is neither convex nor monotonic in  $U(x)$ , there is no guarantee that the worst case of uncertainty is at one of the vertices of  $U(x)$ , according to [3].

This methodology of estimating the worst case of uncertainties is done for each objective function of the problem. Then, the point of the solution obtained in the objective space for the worst case estimate of uncertainty factors is determined, this can be visualized in Fig. (4). Figure (4) distinguishes between the ideal point of real maximizing  $f_{wc}^*(x_0, P)$  and the ideal point of estimated maximizing  $\tilde{f}_{wc}(x_0, P)$ , generated by estimating the worst case with uncertainties in (18).

## VI. SIMULATION RESULTS AND ANALYSIS

For all simulations performed on the standard IEEE 30-bus 6-generator test system, the following parameters were used: population size 100, 0.9 crossover probability and 0.1 mutation probability. The REOA execution stops after 100,000 evaluations of each objective function. The program was written in Java and ran in a 1.60 GHz Core i5-4200u processor with 8 GB of RAM. We used the MOEA Framework, available in [www.moeaframework.org](http://www.moeaframework.org), which is a Java library for the development of multi-objective evolutionary algorithms. All results presented are the best Pareto frontiers found after 30 executions of the algorithm for each scenario. Reference [10]

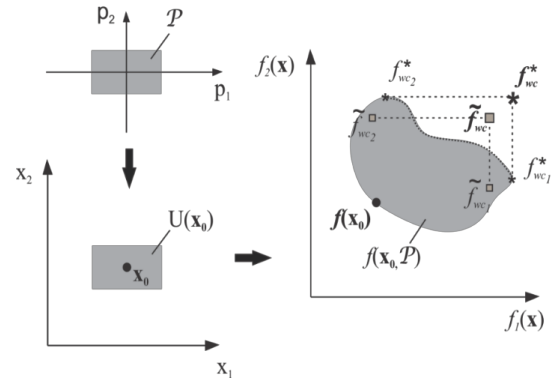


Fig. 4. Worst case estimation for  $f(x_0)$  in multi-objective problems [4].

says the true Pareto optimal set ( $P_{true}$ ) is not explicitly known for real problems. Thus, all Pareto terms used in this paper refers to  $P_{known}$ , in which is the final set of solutions returned by the Multi-objective Optimization Evolutionary Algorithm (MOEA) at termination of execution. More information may be found in [10]. The method used to analyze the results is based on papers found in the literature. For instance, [1] and [8], in which are evaluated the minimum fuel cost and the minimum emission solutions. It was also used a worst case scenario approximation method by samples (WCSA) in [10] to compare the robustness of the ROEA. WCSA consists in generating a number of samples (we used  $2n_v + 1$ ) from a solution disturbing randomly with a value from the uncertainty interval of the decision variables. Next, the worst solution in each dimension of the objective space is used to estimate the worst case scenario.

The experiments were performed in two models of the EED problem, considering and not considering transmission losses of energy. The first model has the term  $X_L$  in (10) to compute the transmission losses of energy.

For the deterministic model without transmission losses, Table (III) and Table (IV) illustrate the minimum fuel cost and the minimum emission found by this paper and compares them with other papers found in the literature. As may be seen, this paper presents better results for the minimum fuel cost and close values considering the minimum emission best result found.

For the robust model without transmission losses, the results found are shown in Table (V) and Table (VI). The values of [1] are not shown because the results are originally presented through graphs. For the minimum fuel cost, REOA found a lower value compared to [8] and the minimum emission is close to the [8]. The Deterministic Pareto Frontier (DPF) and the robust Pareto frontier (ROEA) are illustrated in Fig. (5). It can also be observed in Fig. (5) that all nondominated solutions from REOA are dominated by those from its corresponding deterministic model (DPF). Disturbed Deterministic Pareto Frontier (DDPF) corresponds to the DPF in which all solutions are disturbed using the WCE, which shows that deterministic solutions are more sensitive to uncertainties than

robust solutions. As one can see in Fig. (5), WCSA generated solutions closer to DPF than the REOA.

TABLE III  
MINIMUM FUEL COST FOR THE DETERMINISTIC MODEL WITHOUT TRANSMISSION LOSSES CONSIDERED

	This paper	PSO [8]	NSGA-II [1]
Minimum fuel cost \$/hr	<b>598.454</b>	605.735	600.155
Emission ton/hr	0.2032	0.2064	0.22188

TABLE IV  
MINIMUM EMISSION FOR THE DETERMINISTIC MODEL WITHOUT TRANSMISSION LOSSES CONSIDERED

	This paper	PSO [8]	NSGA-II [1]
Minimum emission ton/hr	<b>0.1883</b>	0.1860	0.19420
Fuel cost \$/hr	635.417	645.300	638.269

TABLE V  
MINIMUM FUEL COST FOR THE ROBUST SOLUTION WITHOUT TRANSMISSION LOSSES CONSIDERED

	REOA	PSO [8]	NSGA-II [1]
Minimum fuel cost \$/hr	<b>601.325</b>	614.571	-
Emission ton/hr	0.2059	0.2065	-

TABLE VI  
MINIMUM EMISSION FOR THE ROBUST SOLUTION WITHOUT TRANSMISSION LOSSES CONSIDERED

	REOA	PSO [8]	NSGA-II [1]
Minimum emission ton/hr	0.1884	<b>0.1866</b>	-
Fuel cost \$/hr	637.338	649.658	-

In this paper, two performance metrics are used to analyze the quality of the sets obtained, hypervolume and Inverted Generational Distance (IGD). The hypervolume metric measures the area dominated by the Pareto solutions, reflecting the dominance of sets but also promotes diverse sets [13]. Briefly, the larger the hypervolume value, the larger the dominated area by the set of solutions in minimization problems. IGD compare two solution sets, in which distances are measured from  $P_{true}$  to  $P_{known}$ . This aims to reduce some of the main problems of this metric in cases in which, for example,  $P_{known}$  has very few points, but they all are clustered together [10]. Thus, the lower the value of the IGD obtained, the better the optimal solutions found. Since we do not have the  $P_{true}$  for the EED problem, we used the DPF as the reference set.

As one can see in Table (VII), DPF has a bigger hypervolume compared to the other fronts, as expected, and its IGD has value zero because its Pareto front is compared to itself. WCSA has the IGD value more close to the DPF, which is expected since this method uses only 13 ( $2n_v + 1$ ) samples. REOA presents a hypervolume lower than DPF and WCSA and its IGD is relative higher compared to WCSA, but lower than DPF. DPF has the worst values for the metrics analyzed in comparison to the other fronts.

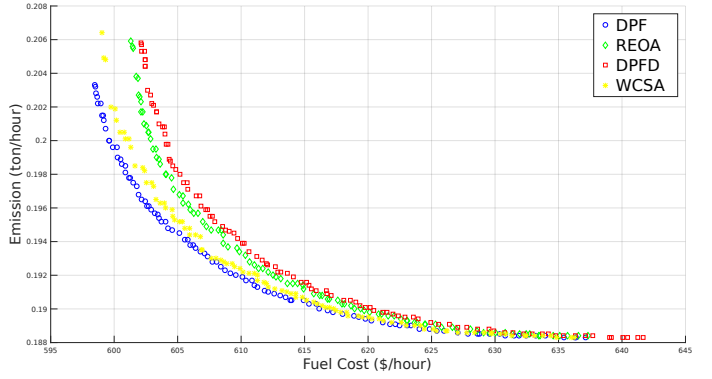


Fig. 5. Comparison of DPF, REOA, DDPF and WCSA without transmission losses considered

TABLE VII  
HYPERVOLUME AND INVERTED GENERATIONAL DISTANCE OF THE FRONTIERS FROM FIG.(5)

	Hypervolume	IGD
DPF	0.78597	0.0
WCSA	0.76865	0.17069
REOA	0.73083	0.53257
DPFD	0.71420	0.67854

In order to verify the quality of WCSA compared to REOA, Fig. (6) shows the number of samples for the WCSA necessary to attain Pareto frontier of REOA. It was necessary to execute WCSA for 10000 of samples to find out a frontier next to REOA Pareto frontier. Table (VIII) shows the hypervolume for the WCSA frontier for 10000 samples and for the REOA.

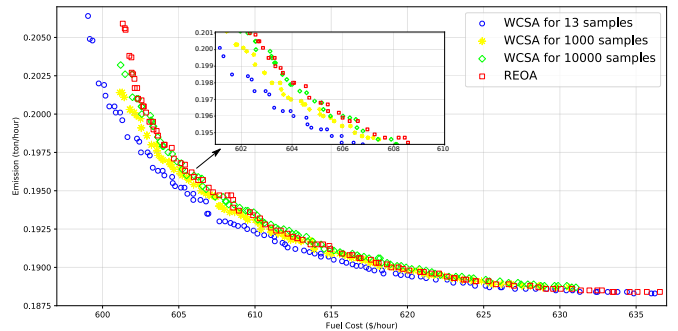


Fig. 6. Comparison of WCSA for 13, 1000, 10000 samples and REOA for the scenario with transmission losses not considered

Based on [12], in which the author analyzes the computational cost of his proposed optimization algorithm considering the number of evaluations of the algorithm, Table (VIII) shows the computational cost for REOA and WCSA. Each objective function has 100000 evaluations (EED has 2 objective functions) for both methods. In REOA, however, each evaluation of a solution has an intern computational cost of 13 ( $2n_v + 1$ ) to estimate the worst case of uncertainty for the EED, which results in 260000 evaluations in total. WCSA has an intern computational cost according to the number of samples. As may be seen, REOA performs 0.13% of WCSA

evaluations for 10000 samples to reach approximately the same Pareto frontier. Table (VIII) summarizes the total number of evaluations used for each of the robust methods.

TABLE VIII  
TOTAL EVALUATIONS REQUIRED TO OBTAIN THE PARETO FRONTIERS FROM FIG.(6) AND HYPERVOLUME

	Hypervolume	Number of Evaluations
WCSA for 13 samples	0.76865	2600000
WCSA for 1000 samples	0.73732	200000000
WCSA for 10000 samples	0.72735	2000000000
REOA	0.73083	2600000

For the deterministic model with transmission losses, Table (IX) and Table (X) illustrate the minimum fuel cost and the minimum emission found considering transmission losses in the system. We found lower values for minimum fuel cost compared to [1] and close to values of [8]. For minimum emission, the values found in this paper is lower than [1] and close to [8].

For the robust model with transmission losses considered, the results found can be seen in Table (XI) and Table (XII). The values found in [1] are not shown because they were originally presented through graphs. For the minimum fuel cost and minimum emission, REOA found a lower value compared to [8]. The Pareto frontiers obtained for both deterministic and robust models considering transmission losses are illustrated in Fig.(7). It can be observed from the Fig.(7) that all the nondominated solutions from the robust model are dominated by those from its corresponding deterministic model. This shows that solutions under uncertainties produce worse results than solutions in which uncertainties in the model are not considered. The DDPF for the respective DPF is also shown, which has solutions that are more sensitive to uncertainties compared to the REOA solutions. WCSA generated solutions closer to DPF than REOA, which shows more robust solutions for the same number of evaluations of WCSA.

TABLE IX  
MINIMUM FUEL COST FOR THE DETERMINISTIC MODEL WITH TRANSMISSION LOSSES CONSIDERED

	This paper	PSO [8]	NSGA-II [1]
Minimum fuel cost \$/hr	607.478	<b>606.541</b>	607.801
Emission ton/hr	0.2008	0.2002	0.21891

TABLE X  
MINIMUM EMISSION FOR THE DETERMINISTIC MODEL WITH TRANSMISSION LOSSES CONSIDERED

	This paper	PSO [8]	NSGA-II [1]
Minimum emission ton/hr	<b>0.1881</b>	0.1864	0.19419
Fuel cost \$/hr	644.642	633.708	644.133

For the scenario with transmission losses considered, the performance metrics are shown in Table (XIII). Results for hypervolume and IGD are very similar to that described in Table (VII). DPF has a bigger hypervolume compared to the

TABLE XI  
MINIMUM FUEL COST FOR THE ROBUST MODEL WITH TRANSMISSION LOSSES CONSIDERED

	REOA	PSO [8]	NSGA-II [1]
Minimum fuel cost \$/hr	<b>610.654</b>	615.042	-
Emission ton/hr	0.2023	0.2017	-

TABLE XII  
MINIMUM EMISSION FOR THE ROBUST MODEL WITH TRANSMISSION LOSSES CONSIDERED

	REOA	PSO [8]	NSGA-II [1]
Minimum emission ton/hr	<b>0.1882</b>	0.1874	-
Fuel cost \$/hr	645.484	637.222	-

other fronts and its IGD has value zero because its Pareto front is also compared to itself. WCSA has the IGD value more close to the DPF, since this method uses only 13 samples. REOA presents a hypervolume lower than DPF and WCSA and its IGD is relative higher compared to WCSA, but lower than DDPF. DDPF has the worst values for the metrics analyzed in comparison to the other fronts.

TABLE XIII  
HYPERVOLUME AND INVERTED GENERATIONAL DISTANCE OF THE FRONTIERS FROM FIG.(7)

	Hypervolume	IGD
DPF	0.64823	0.0
WCSA	0.63229	0.19018
REOA	0.59239	0.62293
DPFD	0.57649	0.84164

Fig. (8) compares WCSA for 13, 1000, 10000 samples to find out how many samples are necessary to approximate the Pareto frontier of REOA. As in Fig. (6), it was necessary to execute WCSA for 10000 samples. In Table (XIV), the hypervolume of the WCSA frontier for 10000 samples and of the REOA frontier can be seen.

Table (VIII) shows that the number of evaluations are the

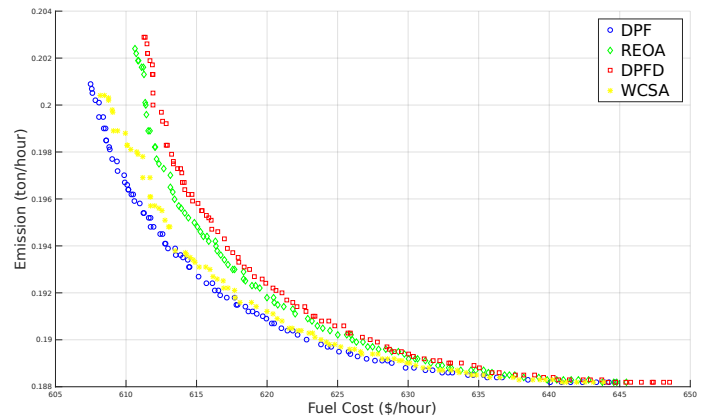


Fig. 7. Comparison of DPF, REOA and DDPF with transmission losses considered

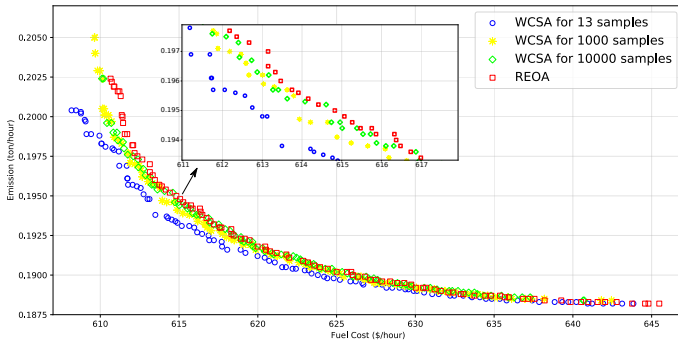


Fig. 8. Comparison of WCSA for 13, 1000, 10000 samples and REOA for the scenario with transmission losses considered

same for both scenarios considering transmission losses and not. Hence, the REOA performs 0.13% of WCSA evaluations for 10000 samples to reach approximately the same Pareto frontier for the scenario which considers transmission losses.

TABLE XIV  
TOTAL EVALUATIONS REQUIRED TO OBTAIN FRONTIERS FROM FIG.(8)  
AND HYPERVOLUME

	Hypervolume	Number of Evaluations
WCSA for 13 samples	0.63229	2600000
WCSA for 1000 samples	0.60454	200000000
WCSA for 10000 samples	0.59826	2000000000
REOA	0.59239	2600000

## VII. CONCLUSION

In this paper, the multi-objective environmental/economic dispatch problem has been solved using NSGA-II and Robust Evolutionary Optimization Algorithm - REOA. The algorithm has been run on the standard IEEE 30-bus system. The deterministic and robust models have been addressed. The REOA uses the worst case estimation of parametric uncertainties. Two scenarios were evaluated: (i) the lossless system and (ii) when transmission losses are taken into consideration. The Pareto frontier for both scenarios was found. In the first scenario, for deterministic model, the minimum fuel cost solution is better than those found in the literature and the minimum emission is close to the best solution found in literature. For the robust model, the first scenario found minimum cost and minimum emission lower than those found in the literature.

For the second scenario, in the deterministic model, the minimum fuel cost and minimum emission solution found is close to the best solution found in the literature. For the robust model, the minimum fuel cost and emission solutions found are lower than those found in literature. The simulation results demonstrate that the solutions for the robust cases are all dominated by their deterministic counterparts. However, if uncertainties are not considered in the optimization model, it is possible that a small disturbance in the decision variables may cause infeasibility of the nominal optimal solution in practice.

Performance metrics, such as hypervolume and Inverted Generational Distance, were also shown. The values found

showed the REOA presented lower computational cost in number of the objective function evaluations compared to another method with a higher cost to produce approximately the same result set.

Thus, it is mandatory to deal with uncertainties in the EED optimization problem considering the standard IEEE 30-bus 6-generator test system. In addition, it is important to develop optimization methods that be able to provide robust solutions and to operate with more reliability in real systems.

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